WATER DROPLET COAGULATION DURING ELECTRICAL DEHYDRATION OF PETROLEUM

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A simple analytical model is constructed for coagulation of emulsion droplets, describing the process of petroleum dehydration by an external electric field.

Some studies of destruction of water—petroleum emulsions by coagulation of droplets have been based on M. Smolukhovskii's theory of colloid coagulation [1, 2], which necessitates solution of cumbersome integrodifferential equations. It would appear desirable to introduce some limitations in the construction of the coagulation model which would achieve full correspondence to the physical formulation of the process being described yet provide sufficient simplicity in the mathematical solution.

The goal of the present study is to construct an analytical model of water droplet coagulation in an emulsion of the inverse type, describing the process of petroleum dehydration in an external electric field.

Let the charged spherical droplets move in a viscous medium within a limited volume under the action of an electric field. The droplet surface will be assumed stabilized by surface-active materials, so that in the future the droplets can be considered rigid and nondeforming [3]. We will divide the droplets into two classes by their size: we distinguish "large" droplets which do not collide with each other, colliding only with "small" droplets (which may also collide with each other). This is possible because of the finite particle path length following from the limited dimensions of the apparatus in question: NESL < 1.

The large droplets change their size by absorbing small ones after colliding with them. Distinguishing this class of large droplets is also desirable from a technological viewpoint, since it is just such droplets which determine the efficiency of the dehydration process. We write $N_0(M_l) = N(M_l(0), 0)$, the initial distribution over mass of the large droplets M_l, $N(M_l(t), t)$, their distribution at some time t. The change in mass of a large droplet is described by the equation

$$\frac{dM_l(t)}{dt} = \int_0^{M_{l\min}} dm\, mN(m)\, \Sigma v_{\mathrm{rel}}. \tag{1}$$

For droplet motion in an electrical field the velocity is an increasing function of droplet size. Given our division of droplets into classes we can assume $R \gg r$ (radii of large and small droplets, respectively), thus v_{rel} is determined by the velocity of motion of large droplets v_l. Thus, for example, if the droplet charge $q \sim R^2$ [4], then $v_l \sim R$, or $v_l = kR$, $k = \pi^2\varepsilon\varepsilon_0E^2/9\eta$. We will limit ourselves to consideration of only this case, although the model can easily be generalized to any other dependence $v_l(R)$.

The question of the capture coefficient is quite complex [5-8]. Capture of small particles by large ones depends on many factors: viscosity of the medium, ratio of the areas, relative velocity, droplet charge, etc. Since at present there is insufficient information on the state of the water—petroleum emulsion, evaluation of the contribution of the various particle precipitation mechanisms is quite difficult. For simplicity, we will assume the coefficient of capture of small droplets by large ones to be a constant value, which with use of the concepts developed herein for real experimental situations should be considered as a parameter to be measured beforehand.

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With consideration of the above, Eq. (1) can be written in the form

$$\frac{dM_l(t)}{dt} = m_\Sigma(t)\alpha\pi R^2 kR,$$

or transforming from radii to masses:

$$\frac{dM_l(t)}{dt} = m_\Sigma(t)\beta M_l(t). \tag{2}$$

Here $\beta = (3/4\rho)\alpha k$. Multiplying both sides of Eq. (2) by $N(M_l, t)$ and summing over masses of large droplets, we obtain a differential equation for the net mass of large droplets

$$\frac{dM_\Sigma(t)}{dt} = \beta M_\Sigma(t)(M_0 - M_\Sigma(t)), \tag{3}$$

$$M_0 = M_\Sigma(t) + m_\Sigma(t) = \text{const.} \tag{4}$$

Solving Eq. (3) for $M_\Sigma(t)$, we again use Eq. (4) and obtain yet another differential equation for determination of $M_l(t)$:

$$\frac{dM_l(t)}{dt} = \beta(M_0 - M_\Sigma(t))M_1(t).$$

The solution of this equation [9] is:

$$M_l(t) = M_{l_{\min}} \frac{C_1 + C_2\beta}{C_1 \exp(M_0\beta t) + C_2\beta} \exp(M_0\beta t),$$

where C_1, C_2 are constants, $C_1^2 + C_2^2 > 0$. For $t = 0$, $M_l(0) = M_{l\min}$. Using the notation $\gamma_0 = (C_2/C_1)\beta$, γ_0 = const, we finally obtain

$$M_l(t) = M_l(0)\frac{1+\gamma_0}{1+\gamma_0\exp(-M_0\beta t)}. \tag{5}$$

The physical meaning of the coefficient γ_0 can easily be established by solution of differential equation (3): $\gamma_0 = m_\Sigma(0)/M_\Sigma(0)$, i.e., γ_0 quantitatively determines the ratio of masses of large and small particles at the moment the process commences.

We will now consider the behavior of the distribution function $N(M_l, t)$. Let

$$f(t) = \frac{1+\gamma_0}{1+\gamma_0\exp(-M_0\beta t)}.$$

At any time t for an arbitrary point $\tilde{M}(t)$ the distribution function can be defined as the ratio $\Delta N(\tilde{M}(t), t)/\Delta M(\tilde{M}(t), t)$. At the initial moment, $t = 0$, the increment

$$\Delta M(\tilde{M}(0), 0) = M_2(0) - M_1(0).$$

Using Eq. (5), we can write

$$\Delta M(\tilde{M}(t), t) = M_2(t) - M_1(t) = \Delta M(0) f(t).$$

Considering that the number of large droplets in the system remains constant (since they do not interact with each other), we write

$$N(\tilde{M}(t), t) = \frac{\Delta N(\tilde{M}(0), 0)}{\Delta M(\tilde{M}(0), 0) f(t)}.$$

The expression $\Delta N(\tilde{M}(0), 0)/\Delta M(\tilde{M}(0), 0)$ is the initial distribution $N_0(\tilde{M}(0))$, $\tilde{M}(0) = M(t)f^{-1}\cdot(t)$. Transforming to the limit, we find that the change in the large droplet distribution function over mass with time can be described analytically:

$$N(M(t), t) = N_0(M(t)f^{-1}(t))f^{-1}(t). \tag{6}$$

Thus the distribution function at any time t depends on the initial system droplet distribution and several physical parameters.

Figure 1 shows results of calculatings with Eq. (6) with a time step of unity, assuming logarithmically normal initial distribution. This distribution was chosen because experimental data on the disperse emulsion composition can most often be approximated by a logarithmically normal distribution [10, 11].

Figure 2 shows the time dependence of total large particle mass, which characterizes efficiency of the dehydration process.

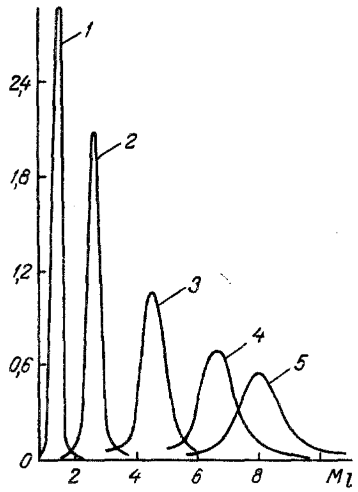

Fig. 1. Time evolution of logarithmically normal distribution for $\gamma_0 = 10$, $M_0\beta = 1$: curves 1-5 correspond to t = 0, 1, 2, 3, 4.

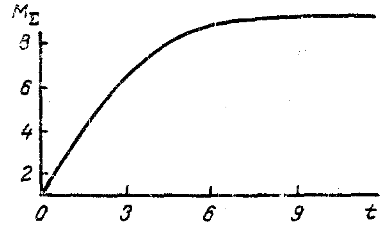

Fig. 2. Change in total mass of large particles in system.

We will now consider special forms of the distribution.

1. Gaussian (normal) distribution with parameters $\bar{M}$, corresponding to mean particle size, and $\sigma/\bar{M}$, corresponding to relative dispersion: $N_0(M) = \frac{1}{\sqrt{2\pi}\sigma} \exp\left(-\frac{M-\bar{M}}{\sigma}\right)^2$.

We apply Eq. (6) to this distribution, i.e. , follow its behavior in time: $N(M, t) = \frac{1}{\sqrt{2\pi}\sigma f(t)}$ $\exp\left(-\frac{Mf^{-1}(t)-\bar{M}}{\sigma}\right)^2 = \frac{1}{\sqrt{2\pi}\sigma f(t)} \exp\left(-\frac{M-\bar{M}f(t)}{\sigma f(t)}\right)^2$.

Consequently, we again obtain a normal distribution with parameters: mean size $\bar{M}_1 = \bar{M}f(t)$ and relative dispersion $\sigma f(t)/(\bar{M}f(t)) = \sigma/\bar{M}$.

2. In a similar manner, the logarithmically normal distribution also maintains its form, only the mean size changing:

$$N(M, t) = \frac{1}{\sqrt{2\pi}\sigma M} \exp\left(-\frac{(\ln M - \ln \bar{M}f(t))^2}{2\sigma^2}\right).$$

NOTATION

$N(M_l)$, $N(m)$, large and small particle distribution functions; M_l, mass of large particles; M_Σ, m_Σ, total mass of large and small particles; t, time; Σ, collision section; α, capture coefficient; v_1, velocity of large particle motion; R, radius; q, droplet charge; ρ, density of dispersed phase particles; M_0, total system mass; β, numerical coefficient; γ_0, constant; ΔN, increment to distribution function; ΔM, change in mass of large droplets; $\sigma/\bar{M}$, relative dispersion of normal distribution; ε_0, absolute dielectric permittivity; ε, dielectric permittivity of dispersed phase; E, electric field intensity; η, viscosity; N_l, number of large particles; S, area of large droplet; L, large particle path length; $M_{l\,min}$, minimum mass of large particle; v_{rel}, velocity of large particles relative to small.

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